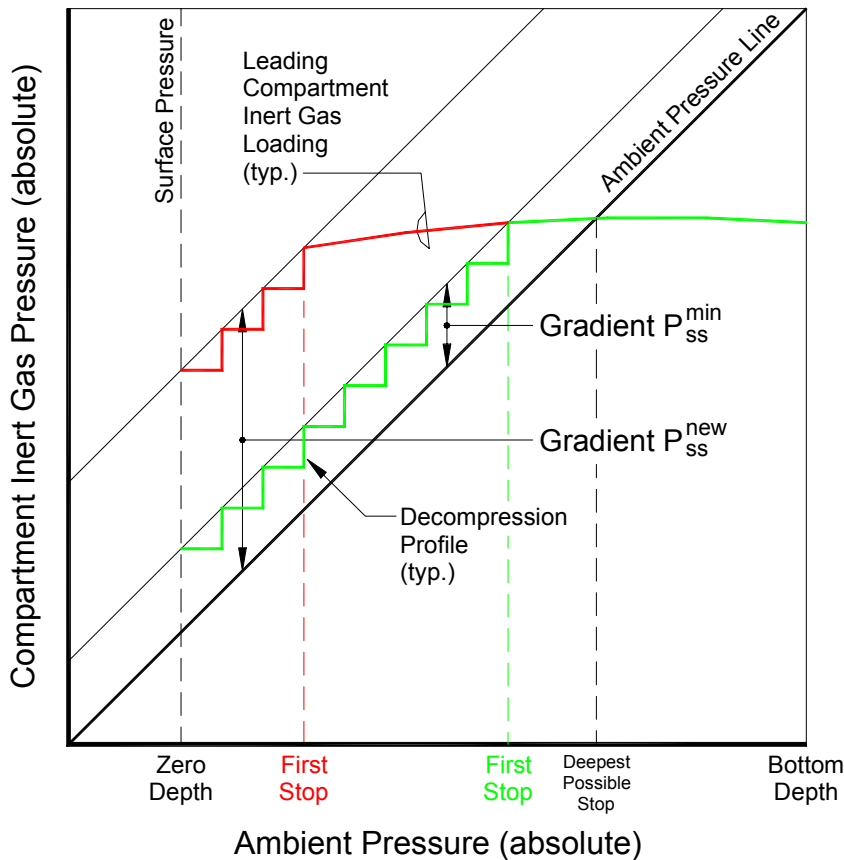


Derivation with Explanation of the VPM Dynamic Critical Volume Algorithm of Yount & Hoffman (1986)

Presented by Erik C. Baker, P.E.

... this paper is prepared with fond memory of the late Professor David E. Yount ...

Pressure Graph: Application of VPM Dynamic Critical Volume Algorithm



$$\text{Bubble Number } N_{\text{safe}} \propto P_{\text{ss}}^{\text{min}}$$

$$\text{Bubble Number } N_{\text{actual}} \propto P_{\text{ss}}^{\text{new}}$$

$$\text{Excess Bubble Number} = N_{\text{actual}} - N_{\text{safe}}$$

Assumptions:

1. The gradient $P_{\text{ss}}^{\text{min}}$ correlates with the constant bubble number N_{safe} and associated released gas volume that can be tolerated indefinitely (i.e. as in a decompression profile from saturation).
2. The body can tolerate N_{safe} number of bubbles regardless of gradient. At a larger gradient $P_{\text{ss}}^{\text{new}}$, the excess released gas volume is proportional to the excess bubble number $N_{\text{actual}} - N_{\text{safe}}$.

Some brief notes about the notation used in this paper:

- A. A capital letter P has traditionally been used by physiologists to denote pressure, either total pressure or partial pressure. Additional description is given by the symbols for individual gases or the use of subscripts. Examples:

P_{N_2} = [partial] pressure of nitrogen

P_{He} = [partial] pressure of helium

P_{amb} = ambient pressure

P_I = inspired [partial] pressure

P_O = initial [partial] pressure

P = [total] pressure

- B. Yount and colleagues used the notation of physicists in their papers. This included a lower case letter p for pressure with descriptive subscripts and superscripts. The same notation is used in this presentation except that a capital letter P is used for pressure (to follow the convention used in diving physiology). Specific Yountian notation is as follows:

P_{ss}^{min} = minimum supersaturation pressure [gradient for bubble formation]

P_{ss}^{new} = new [larger] supersaturation pressure [gradient for bubble formation]

P_{crush} = crushing pressure [gradient for reduction in radius of gas nuclei]

r_o^{min} = minimum initial radius of gas nuclei [probed for bubble formation]

r_o^{new} = new [larger] initial radius of gas nuclei [probed for bubble formation]

- C. A gradient is a difference in pressure which makes it distinct from an absolute pressure. Most of the gradients in this paper use the same notation P as that used for absolute pressures (following the presentation of the original authors). However, I think Bruce Wienke's notation "G" for gradients makes more sense (i.e. a gradient G is independent of the absolute pressure scale whereas a pressure P is not). Accordingly, I use the notation G in a few places where I introduce my own arguments (poetic license!).

Discussion:

The dynamic critical volume hypothesis assumes that the body can eliminate or tolerate N_{safe} number of bubbles and associated volume of released gas for an indefinite period of time. It also assumes that the body can eliminate or tolerate an even greater "critical volume" of released gas for a limited period of time. This critical volume is proportional to the excess bubble number $N_{actual} - N_{safe}$. Gas is entering and leaving the free-phase throughout decompression, so the situation is dynamic. The rate at which the free-phase gas inflates is assumed to be proportional to the product $P_{ss}^{new}(t) \cdot (N_{actual} - N_{safe})$.

The total volume of released gas in the body at any time t should never exceed some critical volume V_{crit} . The decompression criterion is then given by the critical volume equation:

$$\int_0^t P_{ss}^{new}(t) \cdot (N_{actual} - N_{safe}) dt \leq \alpha V_{crit}$$

where α is a constant of proportionality.

The gradient P_{ss}^{new} is assumed to be held constant during the in-water deco time t_{deco} and then it decays exponentially on the surface as the surface interval time $t_{surf} \rightarrow \infty$. Since gradients to drive bubble formation and growth can persist for a period of time after the diver has surfaced, the critical volume decompression criterion includes a phase-volume computation for both the in-water and post-dive (surface) portions of a profile.

Accordingly, the total phase-volume integral is computed in two parts; the in-water portion during the deco time t_{deco} , and the post-dive portion during the surface interval time t_{surf} .

Steps to evaluate the phase-volume integrals in the critical volume equation:

First, factor out constants and separate phase-volume integrals by in-water and surface portions:

$$\alpha V_{crit} = (N_{actual} - N_{safe}) \cdot \left[\int_0^{t_{deco}} P_{ss}^{new}(t) dt + \int_{t_{deco}}^{\infty} P_{ss}^{new}(t) dt \right]$$

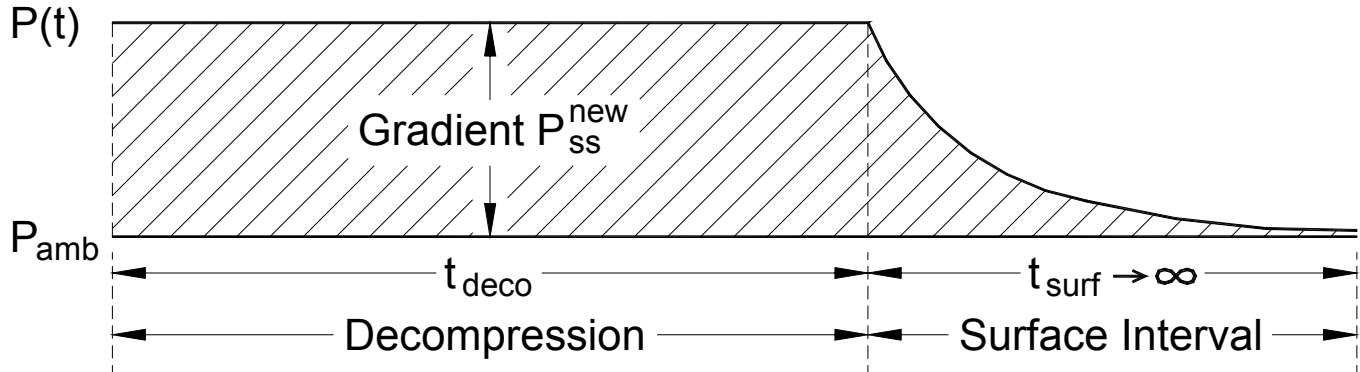
Note that at this point the equation has been set equal to αV_{crit} in order to establish a definite upper limit.

Since P_{ss}^{new} is assumed to be held constant during the deco time t_{deco} , the in-water portion is evaluated as

$$\int_0^{t_{deco}} P_{ss}^{new} dt = P_{ss}^{new} \cdot t \Big|_0^{t_{deco}} = P_{ss}^{new} \cdot t_{deco} - P_{ss}^{new} \cdot 0 = P_{ss}^{new} \cdot t_{deco}$$

In the next step, the phase-volume over the post-dive surface interval must be calculated. This computation is simplified in the original Yount & Hoffman paper by assuming that the diver only breathed a single inert gas such as nitrogen in air during the dive, the diver breathes normal atmospheric air on the surface, and the gradient for dissolved gas elimination is $P(t) - P_{amb}$ instead of $P(t) - P_I$. This last assumption is somewhat conservative and dramatically simplifies evaluation of the integral as will be demonstrated below. In the above notation, $P(t)$ is the partial pressure of dissolved inert gas in a hypothetical tissue compartment as a function of time; P_{amb} is the constant ambient pressure at the surface; and P_I is inspired (alveolar) partial pressure of nitrogen in atmospheric air at the surface.

Phase-Volume Integration



The gas loading as a function of time, $P(t)$, for one inert gas during the surface interval is given by

$$P(t) = P_O + (P_I - P_O) \cdot (1 - e^{-kt})$$

In this case, Yount & Hoffman made the simplifications that $P_I = P_{amb}$ and $P_O = P_{amb} + P_{ss}^{new}$.

Plugging these in and simplifying the gas loading equation:

$$P(t) = P_{amb} + P_{ss}^{new} + [P_{amb} - (P_{amb} + P_{ss}^{new})] \cdot (1 - e^{-kt})$$

$$P(t) = P_{amb} + P_{ss}^{new} - P_{ss}^{new} \cdot (1 - e^{-kt})$$

$$P(t) = P_{amb} + P_{ss}^{new} - P_{ss}^{new} + P_{ss}^{new} \cdot e^{-kt}$$

$$P(t) = P_{amb} + P_{ss}^{new} \cdot e^{-kt}$$

Since the ambient pressure at the surface is constant, the gradient of interest as a function of time, $G(t)$, is given by $P(t) - P_{amb}$. Thus,

$$G(t) = P_{amb} + P_{ss}^{new} \cdot e^{-kt} - P_{amb}$$

$$G(t) = P_{ss}^{new} \cdot e^{-kt}$$

The post-dive (surface) portion is evaluated as:

$$\int_0^{\infty} P_{ss}^{new} \cdot e^{-kt} dt = P_{ss}^{new} \cdot \int_0^{\infty} e^{-kt} dt = P_{ss}^{new} \cdot \left[\frac{1}{-k} \cdot e^{-kt} \Big|_0^{\infty} \right]$$

$$= P_{ss}^{new} \cdot \left[\frac{1}{-k} \cdot e^{-k \cdot \infty} - \frac{1}{-k} \cdot e^{-k \cdot 0} \right], \text{ and since } \lim e^{-\infty} \rightarrow 0 \text{ and } e^0 = 1,$$

$$= P_{ss}^{new} \cdot \frac{1}{k}$$

Thus, after evaluating the in-water and surface phase-volume integrals, the critical volume equation yields:

$$\alpha V_{crit} = (N_{actual} - N_{safe}) \cdot \left[P_{ss}^{new} \cdot t_{deco} + P_{ss}^{new} \cdot \frac{1}{k} \right]$$

$$\alpha V_{crit} = (N_{actual} - N_{safe}) \cdot P_{ss}^{new} \cdot \left(t_{deco} + \frac{1}{k} \right)$$

Note that P_{ss}^{new} is factored out of the total phase-volume integration in order to isolate the variable.

The goal of this mathematical exercise is to be able to solve the critical volume equation for P_{ss}^{new} in terms of calculable parameters. In order to accomplish this, the quantity $(N_{actual} - N_{safe})$ must be expressed in terms of VPM parameters that can be calculated in a decompression program. This leads to the next step in the derivation.

Expressing bubble numbers in terms of calculable VPM parameters:

The VPM primordial (pristine) radial distribution (a continuous distribution function) relating bubble number versus initial radius of gas nuclei, r_o , is given by:

$$N = N_o \cdot \exp\left(\frac{-\beta_o S r_o}{2kT}\right)$$

where β_o is a VPM constant, N_o is a normalization constant, S is the constant area occupied by one surfactant molecule *in situ*, k is the Boltzmann constant, and T is the absolute body temperature which is also assumed to be constant.

The bubble numbers versus initial radii at the gradients P_{ss}^{min} and P_{ss}^{new} , are given by:

$$N_{safe} = N_o \cdot \exp\left(\frac{-\beta_o S r_o^{min}}{2kT}\right)$$

and

$$N_{actual} = N_o \cdot \exp\left(\frac{-\beta_o S r_o^{new}}{2kT}\right)$$

It should be noted at this point that the radial distribution of gas nuclei in humans is not exactly known, so some further assumptions and simplifications are required.

For small values of the exponential argument (linear-small region of the exponential distribution), the above equations can be expanded to simplify the calculation:

$$N = N_o \cdot \exp\left(\frac{-\beta_o S r_o}{2kT}\right) \approx N_o \left(1 - \frac{\beta_o S r_o}{2kT}\right)$$

$$N_{\text{actual}} - N_{\text{safe}} \approx N_o \left(1 - \frac{\beta_o S r_o^{\text{new}}}{2kT}\right) - N_o \left(1 - \frac{\beta_o S r_o^{\text{min}}}{2kT}\right)$$

$$N_{\text{actual}} - N_{\text{safe}} \approx N_o - N_o \left(\frac{\beta_o S r_o^{\text{new}}}{2kT}\right) - N_o + N_o \left(\frac{\beta_o S r_o^{\text{min}}}{2kT}\right)$$

$$N_{\text{actual}} - N_{\text{safe}} \approx N_o \left[1 - \frac{\beta_o S r_o^{\text{new}}}{2kT} - 1 + \frac{\beta_o S r_o^{\text{min}}}{2kT}\right]$$

$$N_{\text{actual}} - N_{\text{safe}} \approx N_o \left(\frac{\beta_o S r_o^{\text{min}}}{2kT} - \frac{\beta_o S r_o^{\text{new}}}{2kT}\right)$$

$$N_{\text{actual}} - N_{\text{safe}} \approx \frac{N_o \beta_o S}{2kT} (r_o^{\text{min}} - r_o^{\text{new}})$$

To further simplify the calculation, r_o^{min} is factored out of the parentheses and a VPM relationship for r_o^{min} is introduced [reference the “Skins” paper by Yount, J. Acoust. Soc. Am. 65(6)1979a]:

$$N_{\text{actual}} - N_{\text{safe}} \approx \frac{N_o \beta_o S r_o^{\text{min}}}{2kT} \left(1 - \frac{r_o^{\text{new}}}{r_o^{\text{min}}}\right)$$

$$r_o^{\text{min}} = \frac{2(\gamma_c - \gamma)}{\beta_o}$$

$$N_{\text{actual}} - N_{\text{safe}} \approx \frac{N_o \beta_o S}{2kT} \cdot \frac{2(\gamma_c - \gamma)}{\beta_o} \cdot \left(1 - \frac{r_o^{\text{new}}}{r_o^{\text{min}}}\right)$$

$$N_{\text{actual}} - N_{\text{safe}} \approx \frac{N_o S (\gamma_c - \gamma)}{kT} \cdot \left(1 - \frac{r_o^{\text{new}}}{r_o^{\text{min}}}\right)$$

At this point, r_o^{min} and r_o^{new} must be expressed in terms of $P_{\text{ss}}^{\text{min}}$ and $P_{\text{ss}}^{\text{new}}$. From the “core” VPM equations:

$$r_o^{\text{min}} = \frac{2(\gamma_c - \gamma)}{\gamma_c \left(P_{\text{ss}}^{\text{min}} - \frac{\gamma}{\gamma_c} \cdot P_{\text{crush}}\right)}$$

$$r_o^{\text{new}} = \frac{2(\gamma_c - \gamma)}{\gamma_c \left(P_{\text{ss}}^{\text{new}} - \frac{\gamma}{\gamma_c} \cdot P_{\text{crush}}\right)}$$

So,

$$\begin{aligned}
& \left(1 - \frac{r_o^{\text{new}}}{r_o^{\text{min}}} \right) \\
&= \left[1 - \frac{\frac{2(\gamma_c - \gamma)}{\gamma_c \left(P_{ss}^{\text{new}} - \frac{\gamma}{\gamma_c} \cdot P_{\text{crush}} \right)}}{\frac{2(\gamma_c - \gamma)}{\gamma_c \left(P_{ss}^{\text{min}} - \frac{\gamma}{\gamma_c} \cdot P_{\text{crush}} \right)}} \right] \\
&= \left[1 - \frac{\left(P_{ss}^{\text{min}} - \frac{\gamma}{\gamma_c} \cdot P_{\text{crush}} \right)}{\left(P_{ss}^{\text{new}} - \frac{\gamma}{\gamma_c} \cdot P_{\text{crush}} \right)} \right] \\
&= \frac{P_{ss}^{\text{new}} - \frac{\gamma}{\gamma_c} \cdot P_{\text{crush}} - \left(P_{ss}^{\text{min}} - \frac{\gamma}{\gamma_c} \cdot P_{\text{crush}} \right)}{P_{ss}^{\text{new}} - \frac{\gamma}{\gamma_c} \cdot P_{\text{crush}}} \\
&= \frac{P_{ss}^{\text{new}} - P_{ss}^{\text{min}}}{P_{ss}^{\text{new}} - \frac{\gamma}{\gamma_c} \cdot P_{\text{crush}}}
\end{aligned}$$

Arranging the critical volume equation so that it can be solved for P_{ss}^{new} :

Now all the previous solutions and simplifications can be plugged back into the critical volume equation:

$$\alpha V_{\text{crit}} = \frac{N_o S (\gamma_c - \gamma)}{kT} \cdot \frac{P_{ss}^{\text{new}} - P_{ss}^{\text{min}}}{P_{ss}^{\text{new}} - \frac{\gamma}{\gamma_c} \cdot P_{\text{crush}}} \cdot P_{ss}^{\text{new}} \left(t_{\text{deco}} + \frac{1}{k} \right)$$

To solve this equation for P_{ss}^{new} , the quadratic formula must be used. To do this, further simplification is required and the equation must be rearranged in the form $ax^2 + bx + c = 0$.

First, the decompression parameter λ is defined :

$$\lambda = \frac{\alpha V_{\text{crit}} \cdot \gamma \cdot N_o \cdot (\gamma_c - \gamma) S}{\gamma_c \cdot kT}$$

$$\alpha V_{\text{crit}} = \frac{\lambda \cdot \gamma \cdot N_o \cdot (\gamma_c - \gamma) S}{\gamma_c \cdot kT}$$

This substitution is made into the critical volume equation and it is rearranged and simplified:

$$\frac{\lambda \cdot \gamma \cdot N_o \cdot (\gamma_c - \gamma) S}{\gamma_c \cdot kT} = \frac{N_o \cdot (\gamma_c - \gamma) S}{kT} \cdot \frac{P_{ss}^{new} - P_{ss}^{min}}{P_{ss}^{new} - \frac{\gamma}{\gamma_c} \cdot P_{crush}} \cdot P_{ss}^{new} \left(t_{deco} + \frac{1}{k} \right)$$

$$\lambda \left(\frac{\gamma}{\gamma_c} \right) \left(P_{ss}^{new} - \frac{\gamma}{\gamma_c} \cdot P_{crush} \right) = \left(P_{ss}^{new} - P_{ss}^{min} \right) \cdot P_{ss}^{new} \left(t_{deco} + \frac{1}{k} \right)$$

$$\lambda \left(\frac{\gamma}{\gamma_c} \right) P_{ss}^{new} - \frac{\lambda \gamma^2}{\gamma_c^2} \cdot P_{crush} = P_{ss}^{new^2} \left(t_{deco} + \frac{1}{k} \right) - P_{ss}^{new} \cdot P_{ss}^{min} \cdot \left(t_{deco} + \frac{1}{k} \right)$$

$$P_{ss}^{new^2} \left(t_{deco} + \frac{1}{k} \right) - P_{ss}^{new} \left[P_{ss}^{min} \left(t_{deco} + \frac{1}{k} \right) + \frac{\lambda \gamma}{\gamma_c} \right] + \frac{\lambda \gamma^2}{\gamma_c^2} \cdot P_{crush} = 0$$

Next, to make the value of the constant **a** in the quadratic formula equal to 1, divide by $\left(t_{deco} + \frac{1}{k} \right)$:

$$P_{ss}^{new^2} - P_{ss}^{new} \left[P_{ss}^{min} + \frac{\lambda \cdot \gamma}{\gamma_c \left(t_{deco} + \frac{1}{k} \right)} \right] + \frac{\lambda \cdot \gamma^2 \cdot P_{crush}}{\gamma_c^2 \left(t_{deco} + \frac{1}{k} \right)} = 0$$

Now the critical volume equation is in the form $ax^2 + bx + c = 0$ and the quadratic formula can be applied:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In this case, the constants are as follows:

$$a = 1$$

$$b = - \left[P_{ss}^{min} + \frac{\lambda \cdot \gamma}{\gamma_c \left(t_{deco} + \frac{1}{k} \right)} \right]$$

$$c = \frac{\lambda \cdot \gamma^2 \cdot P_{crush}}{\gamma_c^2 \left(t_{deco} + \frac{1}{k} \right)}$$

Note that

$$-b = P_{ss}^{min} + \frac{\lambda \cdot \gamma}{\gamma_c \left(t_{deco} + \frac{1}{k} \right)}$$

So, the critical volume equation is solved for P_{ss}^{new} by:

$$P_{ss}^{new} = \frac{b + \sqrt{b^2 - 4c}}{2}$$

where

$$b = P_{ss}^{min} + \frac{\lambda \cdot \gamma}{\gamma_c \left(t_{deco} + \frac{1}{k} \right)}$$

$$c = \frac{\lambda \cdot \gamma^2 \cdot P_{crush}}{\gamma_c^2 \left(t_{deco} + \frac{1}{k} \right)}$$

Note that the $+ \sqrt{b^2 - 4c}$ term yields the largest value for P_{ss}^{new} .

Primary References:

Yount, D.E. and Hoffman, D.C. 1986. On the use of a bubble formation model to calculate diving tables. *Aviat. Space Environ. Med.* 57:149-156.

Yount, D.E. and Hoffman, D.C. 1983. Decompression theory: a dynamic critical-volume hypothesis. In: Bachrach A.J. and Matzen, M.M. eds. *Underwater physiology VIII: Proceedings of the eighth symposium on underwater physiology*. Undersea Medical Society, Bethesda, 131-146.

Varying Permeability Model (VPM) References:

Yount, D.E. 1979a. Skins of varying permeability: a stabilization mechanism for gas cavitation nuclei. *J. Acoust. Soc. Am.* 65:1429-1439.

Yount, D.E. 1979b. Application of a bubble formation model to decompression sickness in rats and humans. *Aviat. Space Environ. Med.* 50:44-50.

Yount, D.E. 1979c. Multiple inert-gas bubble disease: a review of the theory. In: Lambertsen, C.J. and Bornmann, R.C. eds. *Isobaric Inert Gas Counterdiffusion*. Undersea Medical Society, Bethesda, 90-125.

Yount, D.E. 1982. On the evolution, generation, and regeneration of gas cavitation nuclei. *J. Acoust. Soc. Am.* 71:1473-1481.

Yount, D.E., Gillary, E.W., and Hoffman, D.C. 1984. A microscopic investigation of bubble formation nuclei. *J. Acoust. Soc. Am.* 76:1511-1521.

- Yount, D.E. and Hoffman, D.C. 1989. On the use of a bubble formation model to calculate nitrogen and helium diving tables. In: Paganelli, C.V. and Farhi, L.E. eds. *Physiological functions in special environments*. Springer-Verlag, New York, 95-108.
- Yount, D.E., Kunkle, T.D., D'Arrigo, J.S., Ingle, F.W., Yeung, C.M., and Beckman, E.L. (1977). Stabilization of gas cavitation nuclei by surface-active compounds. *Aviat. Space Environ. Med.* 48:185-191.
- Yount, D.E. and Strauss, R.H. 1976. Bubble formation in gelatin: A model for decompression sickness. *J. Appl. Phys.* 47:5081-5089.
- Yount, D.E. and Yeung, C.M. 1981. Bubble formation in supersaturated gelatin: a further investigation of gas cavitation nuclei. *J. Acoust. Soc. Am.* 69:702-708.
- Yount, D.E., Yeung, C.M., and Ingle, F.W. 1979. Determination of the radii of gas cavitation nuclei by filtering gelatin. *J. Acoust. Soc. Am.* 65:1440-1450.
- Yount, D.E. 1997. On the elastic properties of the interfaces that stabilize gas cavitation nuclei. *J. Colloid and Interface Science* 193:50-59.
- Yount, D.E., Maiken, E.B., and Baker, E.C. 2000. Implications of the Varying Permeability Model for Reverse Dive Profiles. In: Lang, M.A. and Lehner, C.E. (eds.). *Proceedings of the Reverse Dive Profiles Workshop*. Smithsonian Institution, Washington, D.C. pp. 29-61.

Additional Decompression and Related References:

- Albano, G. 1970. *Principles and Observations on the Physiology of the Scuba Diver*, English translation: ONR Report DR-150, Office of Naval Research, Department of the Navy, Arlington, Virginia.
- Baker, E.C. 1998a. Understanding M-values. *Immersed*. Vol. 3, No. 3, 23-27.
- Baker, E.C. 1998b. Clearing up the confusion about deep stops. *Immersed*. Vol. 3, No. 4, 23-31.
- Bennett PB, Elliott DH, eds. 1993. *The Physiology and Medicine of Diving*. London: WB Saunders.
- Bühlmann, AA. 1984. *Decompression-Decompression Sickness*. Berlin: Springer-Verlag.
- Bühlmann, A.A. 1995. *Tauchmedizin*. Springer-Verlag, Berlin.
- Cowley, J.R.M., Allegra, C., and Lambertsen, C.J. 1979. Measurement of subcutaneous tissue pressure during superficial isobaric gas counter-diffusion. *J. Appl. Physiol.:Respir. Environ. Exercise Physiol.* 47:224-227.
- Gerth, W.A. and Hemmingsen, E.A. 1976. Gas supersaturation thresholds for spontaneous cavitation in water with gas equilibration pressures up to 570 atm. *Z. Naturforsch.* 31A:1711-1716.

- Hamilton RW, Rogers RE, Powell MR, Vann RD. 1994. Development and validation of no-stop decompression procedures for recreational diving: The DSAT Recreational Dive Planner. Santa Ana, CA: Diving Science and Technology Corp.
- Harvey, E.N., Barnes, D.K., McElroy, W.D., Whiteley, A.H., Pease, D.C., and Cooper, K.W. 1944. Bubble formation in animals. I. Physical factors. *J. Cell. Comp. Physiol.* 24:1-22.
- Hennessy, T.R. and Hempleman H.V. 1977. An examination of the critical released gas volume concept in decompression sickness. *Proc. R. Soc. Lond. B.* 197:299-313.
- Hills, B.A. 1966. A thermodynamic and kinetic approach to decompression sickness. Doctoral thesis, The university of Adelaide, Australia.
- Hills, B.A. 1977. *Decompression Sickness.* John Wiley and Sons, Inc., New York.
- Jablonski, J. and Baker, E.C. 1999. Taking cave diving to the limit. *Immersed.* Vol. 4, No. 3, 29-33.
- Kidd, D.J., Stubbs, R.A., and Weaver, R.S. 1971. Comparative approaches to prophylactic decompression. In: Lambertsen, C.J., ed. *Underwater physiology: Proceedings of the fourth symposium on underwater physiology.* Academic Press, New York, 167-177.
- Kozlov, M.M. and Markin, V.S. 1990. Elastic properties of membranes: monolayers, bilayers, vesicles. *J. Colloid Interface Sci.* 138:332-345.
- Kunkle, T.D. 1979. Bubble nucleation in supersaturated fluids. Univ. of Hawaii Sea Grant Technical Report UNIHI-SEAGRANT-TR-80-01. Pp. 108.
- LeMessurier, D.H. and Hills, B.A. 1965. *Decompression Sickness: A Study of Diving Techniques in the Torres Strait.* *Hvaldradets Skrifter* 48:54-84.
- Maiken, E.B. 1995. "Bubble Decompression Strategies." tek95 Diving Technology Conference.
- Pyle RL. 1996. The importance of deep safety stops: Rethinking ascent patterns from decompression dives. *DeepTech.* 5:64; *Cave Diving Group Newsletter.* 121:2-5.
- Schreiner, H.R. and Kelley, P.L. 1971. A pragmatic view of decompression. In: Lambertsen, C.J., ed. *Underwater physiology: Proceedings of the fourth symposium on underwater physiology.* Academic Press, New York, 205-219.
- Wienke, B.R. 1991. *Basic Decompression Theory and Application.* Best Publishing Co., Flagstaff, Arizona.
- Wienke, B.R. 1994. *Basic diving physics and applications.* Best Publishing Co., Flagstaff, Arizona.
- Workman, R.D. 1965. Calculation of decompression schedules for nitrogen-oxygen and helium-oxygen dives. Research Report 6-65. U.S. Navy Experimental Diving Unit, Washington, D.C.